Refraction of Waves in Two-Dimensions

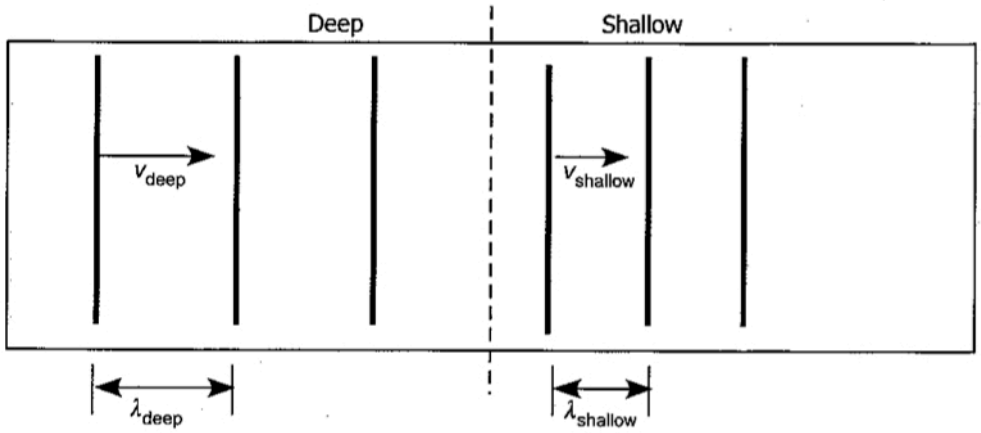
Recall:

- waves travel at different speeds in different media
- when a wave slows down, its wavelength decreases
- when a wave speeds up, its wavelength increases
- the frequency of a wave doesn't change when moving from one medium into another

For example, water waves travel faster in deep water and slower in shallow water.

Waves Moving from One Medium to Another — Head On

The diagram below shows waves initially moving in deep water, parallel to the boundary, that enter directly into shallow water.



As the waves move into shallow water, the frequency does not change, but the speed and wavelength do. The speed of the wave in each medium can be determined using the wave equation:

$$v_{deep} = f \cdot \lambda_{deep} \qquad\qquad v_{shallow} = f \cdot \lambda_{shallow}$$

Solving both equations for frequency, we see that

$$f = \frac{v_{deep}}{\lambda_{deep}} \qquad\qquad f = \frac{v_{shallow}}{\lambda_{shallow}}$$

Since both ratios are equal to the frequency, they must also be equal to each other. Thus,

$$\frac{v_{deep}}{\lambda_{deep}} = \frac{v_{shallow}}{\lambda_{shallow}}$$

This is more commonly written in the form

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

This relationship tells us that if the speed of a wave doubles, the wavelength will also double; if the speed triples, the wavelength triples (and so on).

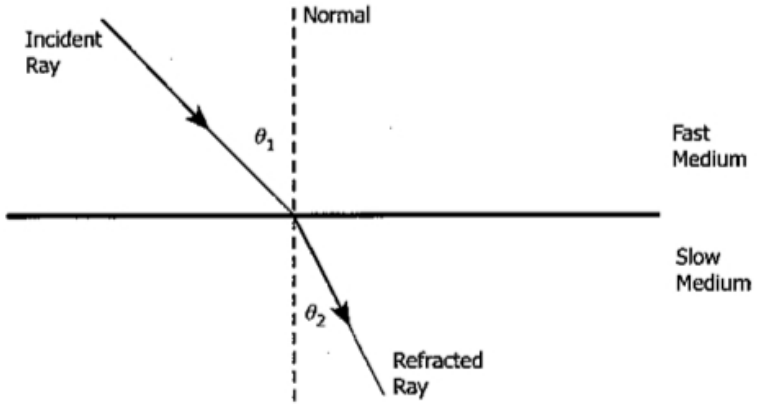
Example 1
A water wave has a wavelength of $2.0\ m$ in the deep end of a pool, and $1.5\ m$ in the shallow end. If the speed of the water wave in the shallow end is $1.2\ m/s$, what is the speed in the deep end?

Refraction — Moving from a Fast Medium to a Slow Medium

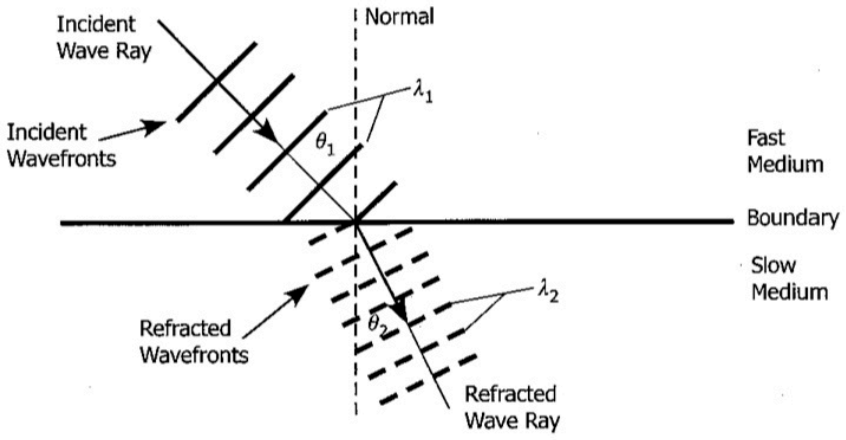
When a wave travels from one medium to another in such a way that it meets the boundary between the two media head on, it simply transmits into the new medium. While the speed and wavelength change, there is no change in the direction of motion.

When the wave meets the boundary at an angle, however, the direction of travel does change. This change in direction of a wave as it passes from one medium to another is called **refraction**.

In general, when a wave travels at an angle into a medium in which its speed decreases, the refracted wave is said to "bend towards the normal." This is illustrated in the diagram below (for the sake of simplicity, only wave rays are shown).



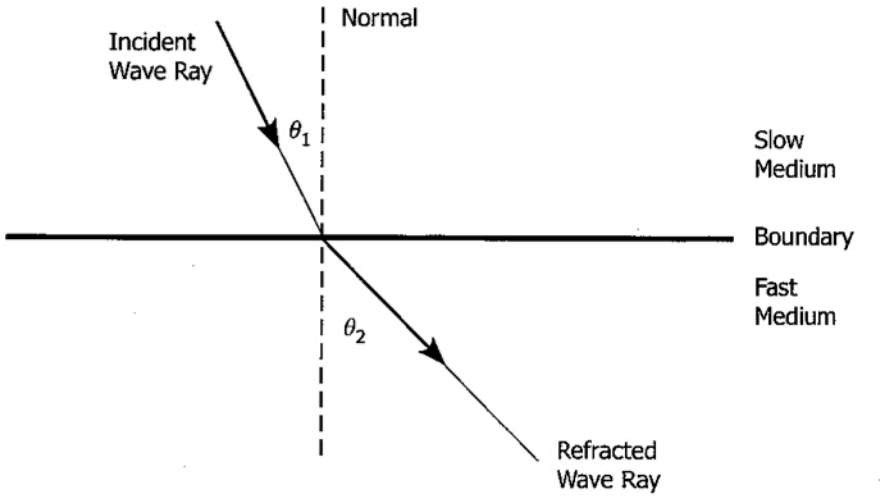
The angle between the incident ray and the normal is called the **angle of incidence** (θ_1). The angle between the refracted ray and the normal is called the **angle of refraction** (θ_2). We can add wavefronts to the diagram to arrive at the following:



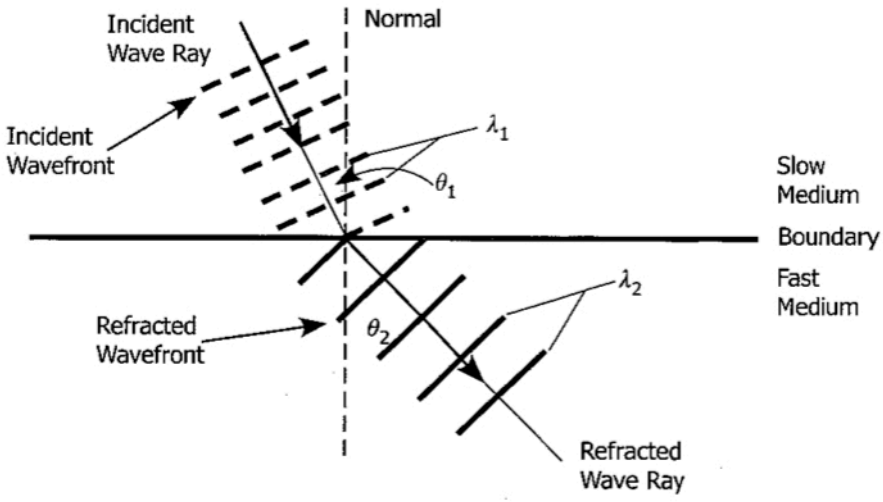
Notice that the wavelength is shorter in the slower medium.

Refraction — Moving from a Slow Medium to a Fast Medium

When a wave travels at an angle into a medium in which its speed increases, the refracted wave is said to "bend away from the normal." This is illustrated in the diagram below.



We can add wavefronts to this diagram as well.



Notice that the wavelength is longer in the faster medium.

The Law of Refraction

The **Law of Refraction**, also known as **Snell's Law**, is a mathematical relationship which relates the speed of the wave in each medium to the angles of incidence and refraction.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

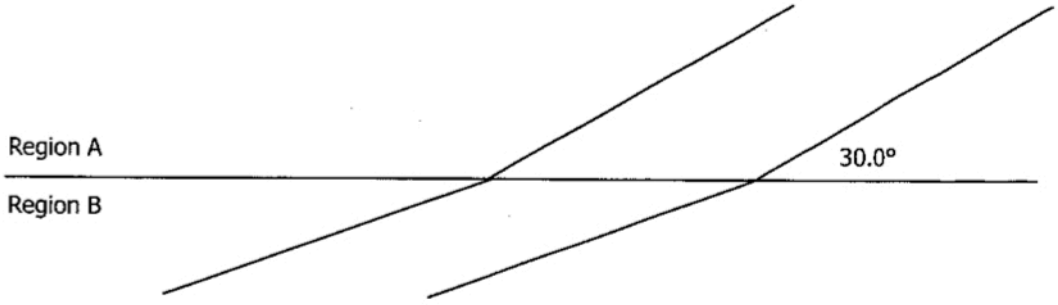
It is worth noting at this point that the angle of incidence can be measured in one of two ways: the angle between the incident ray and the normal, or the angle between the incident wavefront and the boundary. Similarly, the angle of refraction can either be measured between the refracted ray and the normal, or the refracted wavefront and the boundary.

Example 2

The wavelength of a plane wave in the deep end of a ripple tank is $2.00\ cm$, and the frequency is $11.0\ Hz$. Wavefronts strike the boundary of the shallow section of the tank at an angle of $60.0°$ and are refracted at an angle of $30.0°$. Calculate the speed of the wave in deep water and shallow water.

Example 3

A plane wave generator with a frequency of $6.00\ Hz$ creates a water wave with a wavelength of $1.75\ cm$ in region A of a ripple tank. The angle between the wavefronts and the boundary between regions A and B of the tank is $31.0°$.



a) At the point where the incident wavefront hits the boundary for the wavefront on the left, draw in and label the following: the incident wave ray, the normal, the angle of incidence, the refracted wave ray, the angle of refraction.

b) Find the velocity of the wave in region A.

c) On the diagram, measure the wavelength of the wavefronts in region B.

d) Calculate the speed of the wave in region B.

e) Calculate the angle of refraction in region B. Measure the angle of refraction on your diagram. Compare the two values.

f) Which medium is deep water? Explain your reasoning.

Waves Worksheet #6

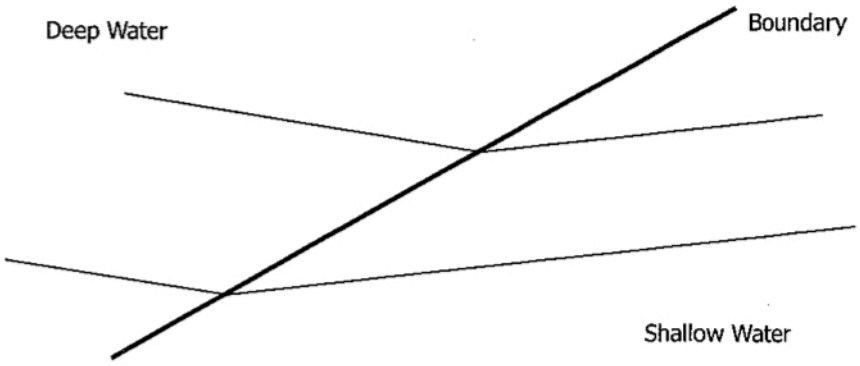
1. A water wave in deep water has a speed of $18.0\ cm/s$ and a wavelength of $2.0\ cm$. In shallow water, the speed of the wave is $10.0\ cm/s$. What is the wavelength of this wave in shallow water? ($1.11\ cm$)

2. A water wave of frequency $10.0\ Hz$ and speed $40.0\ cm/s$ is traveling in deep water. It then moves into shallow water where its speed is $30.0\ cm/s$. The angle of incidence is $30.0°$. Find
   a) the wavelengths in the two media. ($4.0\ cm$, $3.0\ cm$)
   b) the angle of refraction in the shallow water. ($22°$)

3. The velocity of a sound wave in cold air is $320\ m/s$, and in warm air $384\ m/s$. Assume that the wavefront in cold air is nearly linear. What will be the angle of refraction in the warm air if the angle of incidence is $30.0°$? ($37°$)

4. An earthquake P wave traveling at $9.0\ km/s$ strikes a boundary within the earth between two kinds of material. If it approaches the boundary at an incident angle of $47°$ and the angle of refraction is $27°$, what is the speed in the second medium? ($5.6\ km/s$)

5. In the diagram below, draw in the following: the incident wave ray, the normal, the angle of incidence, the refracted wave ray, the angle of refraction. Measure and state the measurements of the wavelengths and the angles of incidence and refraction.



6. A wave travels 0.75 times as fast in shallow water as it does in deep water. What will the wavelength of the wave in deep water be, if the wavelength is $2.7\ cm$ in shallow water? ($3.6\ cm/s$)

7. A ripple tank wave passes from a deep to a shallow region with an angle of incidence of $60°$ and an angle of refraction of $45°$. What are the ratios in the two media of
   a) the wavelengths? (1.2)
   b) the velocities? (1.2)
   c) the frequencies? (1.0)

8. When a straight periodic wave crosses a boundary between deep and shallow water, the following observations are made: ten wavelengths cross the boundary every 5.0 s, and the distance across three wavefronts is 24.0 cm in deep water and 18.0 cm in shallow water. Calculate the velocity of the wave in deep water and in shallow water. ($24\ cm/s, 18\ cm/s$)